Chapter 10

Bellasanta Ferrer: In a (functional) Relationship?

Bellasanta Ferrer, a native of the Philippines, has lived in Chicago, Illinois since 2001. A former geologist, Bellasanta has been a middle school science teacher at Haines Elementary in Chinatown, Chicago since 2012. Bellasanta received her B.S. in Geology from the University of the Philippines. As part of the AUSL's first cohort, she earned her MAT in Elementary Education, with middle school endorsements in mathematics, general science, and physical science, from National Louis University in 2003. This year, 2014, Bellasanta is in the first cohort of the MSU-WIPRO Urban STEM & Leadership Fellowship program.

In a (functional) relationship? (aka Relationship between perimeter & area)

Grade Level: 6th-grade (middle school)

Content Area Topic: Perimeter and area, a non-constant relationship

Content Area Standard(s):
- CC6.EE: Expressions and Equations 9. Represent and analyze quantitative relationships between dependent and independent variables
- CC6.G: Geometry 1. Solve real-world and mathematical problems involving area, surface area, and volume.

Mathematical Practices:
- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision

Learning Objective(s):
- To form at most 4 different shapes with the same perimeter using string or pipe cleaner
- To estimate area (square cm) of the shapes while attending to precision
- To describe relationship between a shape's area and a constant perimeter (explicitly, quantitatively, and concretely)
- To construct viable arguments for the dog area shape proposal and to critique the reasoning of others

Suggested Time Allotment:
Two 45-minute sessions or one 90-minute block

Sequence in Learning:
Requisite lessons:
A. Investigating the relationship between the perimeter and area of squares that increase in side lengths by one centimeter unit. This investigation is part of the algebra unit on algebraic thinking and functions including multiple representations of functions i.e. in a context, a table of paired values, as a formula, and as linear and non-

linear graphs.
B. Estimating perimeter and area of an irregular shape, their own palm. The estimating strategies here can be used for the next lesson.

Requisite knowledge/CCSS Math:
A. Measurement and Data -
- 1. Measure and estimate lengths in standard units: Measure the length of an object by selecting and using appropriate tools i.e. rulers, yardsticks, meter sticks, & measuring tapes
- 2. Geometric measurement:
- a. Understand concepts of area and relate area to multiplication and to addition
- b. Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

B. Geometry -
- Classify two-dimensional figures into categories based on their properties: Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category (e.g. all rectangles have 4 right angles and squares are rectangles, so all squares have 4 right angles and vice versa)

Subsequent lesson:

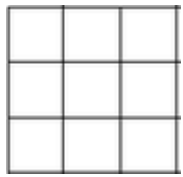
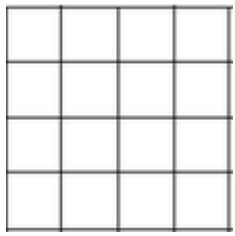
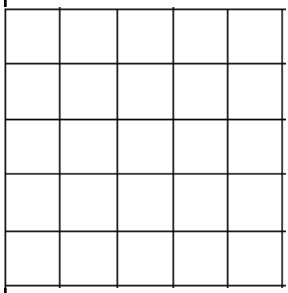
A. Optimal fenced-in rectangular brick patio (a landscaping design project proposal simulation). This follow-up real-world design project aims to provide students the opportunity to apply the concepts of perimeter and area and their non-constant relationship. Students will create or design a fenced-in rectangular brick patio as specified by a client of a landscaping company. This experience will not only contribute to a general understanding of the relationship between shape, size, and economics but will also extend students' earlier work on pattern variation in the perimeter and area of rectangles. This experience will likewise lay a foundation for a further examination of surface area in geometry and calculus.

Materials & Resources Needed:
Each group needs:
- 48-cm string or pipe cleaners
- ruler
- scissors
- grid paper (with 10mm or 20mm primary grid and 2mm sub-increment grid http://customgraph.com/samples/customgraph-engineering.pdf)
- data table or spreadsheet on paper or online

Lesson Activities & Sequence:

Activation of Prior Knowledge: (10-15 minutes)

1. Review previous investigation (Growing Squares) on the relationship between perimeter and area of squares that increased in side lengths by 1 cm.

Illustrated or actual tiled squares	Side length (cm)	Perimeter (cm)	Area (square cm or cm^2)
	1	4	1
	2	8	4
	3	12	9
	4	16	16
	5	20	25
	n	4n	n^2

Review generalizations based on this investigation (students can explain in words, algebraically, physically, and/or visually)

The perimeter of a square ...

The area of a square…

2. Review strategies used to estimate area of students' own palm

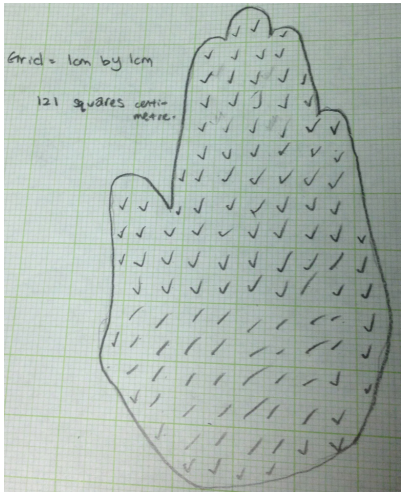


Image from http://rongshanphysics.wordpress.com/

3. Launch of investigation/challenge: In a relationship?
What does it mean when someone is in a relationship like in FB?
What could it mean when something is in a relationship (like dependent and independent variables?

Connect back to the previous lesson that modeled the relationship between perimeter and the side length of a square and the relationship between the square's area and its side length.

Challenge: Today, your team will help solve the problem of two people in a relationship. They have a garden that they want to use for an enclosed play area for their dog, for growing herbs, and for outdoor entertainment. They want you to suggest the best shape that will enclose the most space for their dog's play area using their 12-meter fencing material.
You have 10-15 minutes to plan and create your shapes and use the area estimation strategies from yesterday's lesson to find the area of your shapes. Record your work on paper (student- or teacher-made) or electronically. Each team will present and argue in favor of the proposed dog play area. Be ready to look for what each team did well and to critique each others' reasoning. Any questions?

Group students according to self-perceived level of understanding of the two previous lessons on the perimeter and area of the growing squares and palm area estimation.
Review team members' responsibility (materials manager, group facilitator/time keeper, spokesperson, recorder/procedure captain)

Each group can decide which materials/technology to use: a) string or the pipe cleaner, b) grid paper (with 10mm or 20mm primary grid and 2mm sub-increment grid), c) paper or online data table or spreadsheet, d) student-made or teacher-made data table, e) to present their team's proposal, and f) provide feedback and critique

Proficiency:
The rubric below will be given in advance and used for team self-assessment.

	Incomplete &/or incorrect	One section Complete but missing	Complete & correct	Above & beyond
Shapes	less than 4 shapes	4 shapes but	4 different shapes with same perimeter	+systematic increase in area
Area	incorrect or incomplete estimates	estimates area but	appropriately estimates area of all 4 shapes	+explains precision options
Area & perimeter relation-ship	incomplete or unclear description P/A non-constant relationship	completely describes P/A non-constant relation-ship but...	clearly/ correctly describes the non-constant P/A relationship	+describes relationship in 3 different ways
evidence-based argument or proposal	incomplete or unclear proposal	presents proposal for fenced-in dog area but...	completely & correctly argues for proposed fenced-in dog area based on evidence	+uses appropriate technology, data, or observations as evidence to compliment & critique the proposal of other teams
Team work	some members were off-task for parts of the challenge	members perform roles but...	members perform roles as expected	+members model team & individual accountabil-ity

Best shape for enclosed dog play area

Shape	Perimeter (cm or mm)	Area (square cm or square mm)

Feedback

Teachers As Learners:
Availability of different materials to use, paper, or physical objects, or computation to explore different shapes and find its area.

Some were confused over whether the area that was being created was for the dog only or to include the other two, herb garden and outdoor entertainment space. One way to avoid this confusion would be to start with just the dog play area then add the other spaces as an extension of the main lesson.
Intentional grouping, scaffold visual or ESL learners by providing pictures (dog, garden) and screenshot of FB. Including a screenshot of the "in a relationship" status page in Facebook might help students make the connection to the lesson title and the Facebook social media.

Lesson could be easy to adapt to different grade levels, abilities, and learning modalities including being an English language learner.
Activated prior knowledge
Allowed students to choose the materials to use in their exploration and design.
Had students present and critique each others' process and proposal to the class.

Elements of Pretty Good Practice:
- Reviewed previous lessons to assess and activate prior knowledge
- Different students had a choice on what materials to use and how to use them. The pipe cleaner allowed students to recreate their shapes as needed to correct 'mistakes.' The other groups loved how the multi-grid graph paper facilitated the estimation of the shapes' area.

- Teacher moved between groups, answering questions, observing
- Allowing students to group according to prior knowledge
- Teacher expectations laid out through use of rubric
- Students evaluated each other's work rather than by the teacher

Modifications and Adaptations:
- Extend this lesson by providing more complex problems such as: Redesign of school classroom or school garden.
- Students were given graph paper with 10mm or 20mm primary grid and 2mm sub-increment grid
- Include connecting cubes manipulatives
- Students design their own questions and give them to others
- I could have the students create shapes with specific lengths and widths so they can relate to the previous lesson on squares with changing side lengths
- Data can also be plotted in a length versus area graph for another representation

Questions Arisen:

Content

Was the proposal about finding the dog play space with the most area using the 12-meter fencing materials or to include the other two spaces, herb garden and outdoor entertainment?

Pedagogical approach
- Alternative questions can be designed for using all three areas. You can rephrase questions to assess different ideas.
- How to group? Intentionality?
- How would the use of technology change how the lesson is delivered?

Technology
- Are there applications to draw figures and calculate areas?
- Does the use of technology, i.e. Geogebra, allow for more inquiry and argumentative discourse?
- *Peer Feedback:*
- They suggested to stay in teacher mode throughout the teaching demo session. They prefer that no explanations of the teacher's pedagogy are given while the demo is ongoing.

Bibliography:
- Ferrer, B., Hunter, B., Irwin, K., Sheldon, M., Thompson, C., and Vistro-Yu, C. (2001). By the Unit or Square Unit? Mathematics Teaching in the Middle School. National Council of Teachers of Mathematics (NCTM). 7, 132-137

- Gavin, K., Belkin, P., Spinelli, A., and St. Marie, J. (2001), Navigating through Geometry in Grades 3-5. National Council of Teachers of Mathematics.
- Nichols, E., et al. (1985), Holt Mathematics-Grade 7. New York: Holt, Rinehart & Winston.

Related Resources/Ideas: